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APPLICATION OF SKEWNESS IN PASSING OF ARCH-GARCH MODEL COMMENCE FOR CURRENCY PORTFOLIOS

The paper uses Coskewness as risk measure, average return and gives detail of efficient skewness (gamma) of a diversity of currency portfolios. This paper also applies ARCH-GARCH model, significant properties of GARCH allow to efficient modeling financial time series having obese conclusions. Then, we connect Coskewness with ARCH-GARCH models to optimize currency portfolio. To conclude, an empirical study of ten currency portfolios from Pakistan currency exchange market is performed and all the results suggest that Coskewness can better characterized the risk-adjustment and average variance and the performance of ARCH-GARCH model is better than that of ARIMA model in portfolio optimization.

Key words: *Currency portfolios, Coskewness, ARCH and GARCH models, Pakistan Rupee.*

Мунієб Ахмад, Юсаф Алі Хан, Іртаза Іштіак, Мухаммед Масуд. Використання асиметрії у побудові моделі ARCH-GARCH під час аналізу валютних портфелів

У статті представлено використання співасиметрії як інструменту оцінки ризику, середніх значень статистичної моделі та для представлення розширеної інформації щодо ефективної асиметрії (гамма) множини валютних портфелів. У дослідженні також використовується модель ARCH-GARCH, оскільки широкі можливості моделей типу GARCH дозволяють ефективно моделювати фінансові часові ряди із суттєвими залишками.

У цій статті представлено дослідження зміни пропорцій валютного портфеля на Пакистанській валютній біржі для збереження його профіцитності.

Метою статті є дослідження оптимізації валютного портфеля в межах середньодисперсійного аналізу, асиметрії та співасиметрії у рамках моделі ARCH-GARCH для визначення оптимальної структури валютного портфеля, а також обґрунтування доцільності інвестицій у валютні торговельні портфелі пакистанських інвесторів як альтернативи інвестиціям у конкретну валюту.

На підставі представлених розрахунків обґрунтовано шляхи оптимізації валютного портфеля з урахуванням ризиків на основі поєднання дослідження співасиметрії досліджуваних параметрів та моделі ARCH-GARCH. Для оцінки з підвищеною точністю ризиковості валютного портфеля у статті застосовується співасиметрія для визначення ризиків, середніх значень статистичної моделі та для отримання розширеної інформації щодо ефективної асиметрії (гамма) множини валютних портфелів. Співасиметрія та модель ARCH-GARCH максимально повно враховують вагу пропорцій валютного портфеля та варіації залишків моделі, що підвищує ефективність прогнозу валютних вигод. Обґрунтовано переваги використання моделей ARCH-GARCH для оптимізації портфеля перед моделлю ARIMA. При цьому доведено, що найбільш оптимальний валютний портфель формується за умови поєднання дослідження співасиметрії та моделі ARCH-GARCH.

Проведено емпіричне дослідження десяти валютних портфелів пакистанського валютного ринку, в результаті чого доведено, що співасиметрія краще характеризує коригування ризику та середню дисперсію, а для оптимізації портфеля ефективніше використовувати модель ARCH-GARCH. На підставі цього запропоновано використання дослідження співасиметрії та моделі ARCH-GARCH у майбутніх дослідженнях, зокрема для визначення оптимального валютного портфеля із середньою прибутковістю та коригуванням ризику на основі портфеля акцій та валюти.

Ключові слова: *валютний портфель, модель ARCH-GARCH, Пакистанська рупія, співасиметрія.*

Formulation of the problem. Markowitz, 1952, sets up the foundation of the expansion of present portfolio theory. He takes the combination of risk and returns to construct the optimal portfolio and suggests the Mean-Variance model, with expected return and estimated portfolio risk characterized portfolio return. Coskewness is much like covariance. It is determined of a security's threat connected to marketplace risk. It was first used to investigate hazard in stock market ventures by Kraus and Litzenberger in 1976, then [4] using the non central-t distribution and proposed a new technique for confined the time-series deviation in skewness. Lean Yu, 2008, stated skewness is sound recognized technique to choose a portfolio as, has established that for three broadly exchanged outside trades ($/€, $/£ and $/¥) mean–variance skewness is a productive method for settling the exchange off portfolio determination issue. The consolidation skewness in financial specialist's group choice origin significant transform for development an ideal assortment as various mixes of speculator's inclinations lead to ideal portfolios [7].

Analysis of recent research and publications. According to Chonghui Jiang [3], the anticipated impacts of skewness on cross-sectional money abundance profits are altogether optimistic.

A model integrating Coskewness is supportive in describing the cross-sectional peculiarity of investment profits. Coskewness can't model the instantaneous standard deviation as a random process to its own. Modifications in the variation or volatility in excess of time can origin troubles while modeling time series with conventional techniques similar to ARIMA. Specifically, we explain how the average error range can be replicated as an auto-regressive progression; these models are called ARCH and GARCH.

The ARCH or Autoregressive uncertain Heteroscedasticity technique presents a way to model an alteration in variation in a time series toward the time reliant, such as growing or declining volatility, Fixed the model by counting delay uncertain volatility terms, generating GARCH models [1]. GARCH permits the process to sustain alterations in the moment dependent volatility, such as rising and falling instability in the similar series. Engle (1982) proposed the ARCH model; these weights are constraints to be projected. Engle's ARCH model there by permits the facts to conclude the most excellent weights to utilize in anticipating the variation. The investigation of ARCH and GARCH models give a computable point for portfolio examination (Engle, 2008). Gabriel [6] & Nawaz Ahmad, 2016, the ARCH and GARCH models are extremely famous to choice for the various sort of portfolios with best gainfulness returns and less risk as [6] depicted the GARCH models have been likewise assessed dependent on their estimating capacity of things to come returns. The GARCH models have been additionally assessed dependent on their anticipating capacity of things to come returns (Oana Predescu, 2011). GARCH means to limit mistakes in estimating by representing blunders in earlier determining and, accordingly, upgrading the precision of progressing forecasts (Kenton, 2018).

The currency exchange market is the biggest market on the planet. It's a method to get one currency and sell another simultaneously, by exchanging currency exchange portfolios (Josephson, 2019). Currency portfolios are significant for each field of business the Affiliation Money Portfolios, developed suggestion, affiliation tenet withdrawal; fortify mean-variance productivity for cash speculators [2]. The remote currency exchange market is principal and mainly fluid market of world, with a normal day by day capacity profusion of $6 trillion (Kuepper, 2020). Wen and Yang [5] suggest, the majority investors are risk unwilling, whereas main entrepreneurs are risk forbearing afterward they utilized the coefficient in the GARCH-M model as the determinant for risk approach. The ARCH model for estimation of time fluctuating skewness and kurtosis firstly was used by Hansen [8]. In our study, we evaluated most likely skewed and highly-tailed allocations for the reaction. Conventional time's series models have been created as vibrant models for the mean and the variance. The ordinary formation of ARCH and GARCH models are then obviously observable in the provisional mean and the uncertain variance [11].

To account for both excess kurtosis and asymmetries, GARCH models with a large number of alternative asymmetric conditional densities have been employed, for example, the asymmetric stable density [12] and the Gram-Charlie Expansion [13]. Hansen [8] was the first to allow for time varying skewness and kurtosis by extending the ARCH framework. Jondeau and Rockinger [9] find the presence of time varying skewness and kurtosis in daily but not weekly data. This is consistent with the empirical fact that excess kurtosis diminishes with temporal aggregation [14].

The purpose of the article. This paper examines currency portfolio optimization within a mean-variance-systematic skewness and ARCH-GARCH structures. Before our study no appropriate research made particularly concerning the assortment of currency exchange portfolio regarding Pakistani Rupee, the rationale of this study is to uncover the counter of these research queries. First, why for Pakistani financiers are enhanced to invest in currency trade portfolios investors as a replacement for investing in a particular currency? Second why the assortment of currency exchange portfolio is significant?

Presentation of basic material of the research. Assume m-risky resources accessible in the venture set through a feature revisit vector, r a risk-free currency exchange market through profit r_f. A portfolio p of m-risky currency exchange markets is a vector $v_p = (v_1, v_2, \dots\dots, v_p)^1$, wherever v_{pi}(j=1, 2… m) is the share of the portfolio invested in currency exchange market j and the great script l symbolized the invert maneuver. Therefore, portfolios profit is

$$r_p = r_f + v_p^1(r - 1mr_f) \tag{1}$$

Anywhere 1m is m-column vector among every one of basics equivalent to single, anticipation plus variation of assortment profits as follows:

$$E(r_p) = r_f + v_p^1(E(r) - 1mr_f) \tag{2}$$

$$\sigma_p^2 = v_p^1 \Sigma v_p \tag{3}$$

Where Σ locates for the covariance matrix m-risky return, along with non-singular. Marketplace index with profit represented as r_w inconsistency denotes σ_w^2, skewness denoted while $\mu_w^3(\mu_w^3 = E[(r_w - E(r_w))^3]$ the co-efficient of skewness of assets j is denoted by $\beta_1 = Cos(r_j, r_w)$ is distinct like the moment among the lowered currency exchange market profits and marketplace indicator profit.

$$Cos(r_j, r_w) = E[(r_i - E(r_i)(r_w - E(r_w))^2] \tag{4}$$

There are likewise other elective proportions Coskewness, the Coskewness while characterized in Equation.4 accompanying two motives. It contains reasonable financial understanding, speculators be especially keen on sort of Coskewness in portfolio determination. Instinctively, it quantify speaks to minimal commitment of a money trade market skewness for marketplace incomes, and accordingly profits on cash trade markets among optimistic Coskewness are more emphatically slanted than showcase proceeds. Along these lines, expanding the loads of money trade markets through optimistic Coskewness within portfolio is able to facilitate the portfolio's skewness. Second, a portfolio's Coskewness is only slanted normal of Coskewness of individual currency exchange showcases in assortment. Along these lines, the Coskewness of a portfolio is direct in selection loads, guarantees to our model considered diagnostically well-mannered.

Lemma.1. Signify the tributary of profits greater than a quantity phase following take away the average return over the period as $h_w = (r_w - E(r_w))^2$. A currency exchange market's Coskewness is the covariance linking currency exchange market's income and h_w, in favor of currency exchange market j, we have:

$$\mathrm{Cos}\ (r_j, r_w) = \mathrm{cov}\ (r_j, h_w),\ (j= 1, 2 \ldots m) \tag{5}$$

Proof: The tributary of profits greater than a quantity phase following take away the average return over the period is denoted by $h_w= (r_w - E\ (r_w))2$ where $r_w$characterizes at hand put on a marketplace catalog. For several currency exchanges market Cos (r_j, r_w), the covariance amongst its profit r_j and the tetragon degraded marketplace profit h_w specified by:

$$\mathrm{Cos}\ (r_j, r_w r_j, r_w) = E\ ((r_j - E(r_j)(h_w - E(h_w))) \tag{5.1}$$

$$= E\ ((r_j - E(r_j)h_w - E((r_j - E(r_j)E(h_w)) \tag{5.2}$$

$$= E\ ((r_j - E(r_j)h_w = E((r_j - E(r_j)(r_w - E(r_w))^2) \tag{5.3}$$

The right-hand side of Eq. (5.3) is basically the Coskewness of profits on currency exchange marketplace r_j with market returns Cos (r_j, r_w) .As a result,

$$\mathrm{Cos}\ (r_j, r_w) = \mathrm{Cov}\ (r_j, h_w) \tag{5.4}$$

Note that hw determines the inconstancy of marketplace profits.

Lemma 1 clarifies decidedly slanted cash trade showcase is equipped for supporting against stuns to market instability, as the money exchange market profit will in general increment as the market turns out to be increasingly unpredictable. Since cash trade market's Coskewness shifts with markets skewness, numerous investigations center on the money trade market's Coskewness, which is a normalized proportion of Coskewness, so as to evade scale issues [15]. The Coskewness of cash exchange market j is the money trade market's Coskewness standardized by the market-skewness, as illustrated by Kraus and Litzenberger (1976) showcase skewness can be express as:

$$\beta_j = \mathrm{Cos}\ (r_j, r_w)/\ \mu_w^3 (j = 1, 2 \ldots m) \tag{6}$$

Based on Lemma 1, the Coskewness of cash trade showcase j is given by $\beta_j =$ $= \mathrm{Cos}\ (r_j, h_w)/\mu_w^3$. This proposes a money trade market's Coskewness estimates the affectability of the cash trade market's arrival to changes in the instability of market returns. Therefore, a cash trade market's Coskewness (β) matches the idea of beta (β*), which gauges the affectability of the money trade market's arrival to transform market proceeds. Indicate the Coskewness vector of m-dangerous cash trade markets as $\Lambda = (\beta_1, \beta_1, \ldots, \beta_n)^1$. At that point, portfolio v_p's Coskewness can be communicated as $v_p^1 \Lambda$. The mean-change Coskewness model is given as follows:

$$\min_{v_p \epsilon \mathcal{R}^n} \sigma_p^2 = v_p^1 \Sigma v_p \tag{7}$$

$$\text{s.t} \qquad \mu_p = r_f + v_p^1 (r - 1mr_f) \tag{8}$$

$$v_p^1 \Lambda = \beta_p \tag{9}$$

Where μ_p and β_p are the pre-specified portfolio's predictable profits and Coskewness, correspondingly. There are a number of financial inferences of model. With no Eq. (9), depositors' portfolio choice depends exclusively on the uniqueness of currency exchange market proceeds and discrepancy. With Eq. (9), the financier currently also believes a currency exchange market's aptitude to be cautious beside unanticipated distress to marketplace instability, and précised by its Coskewness.

By Lemma 1, financier choose currency exchange markets among affirmative Coskewness further individuals by pessimistic Coskewness, as adding the seas sets to a portfolio decreases the resulting portfolio's possibility of great depressing proceeds with augment the chances of intense optimistic income. To realize preferred affirmative portfolio Coskewness, the financier ought to deposit a depressing β_p when the marketplace skewness is pessimistic. On the other hand, the financier ought to set an optimistic β_p but the marketplace skewness is affirmative. Therefore, the proficient portfolios resolute through our model (mean-variance Coskewness (MVC) portfolios) are instantly connected to marketplace situations. Naturally, throughout phase of market downward spin through elevated instabilities (the case of negatively-skewed investment profits) currency exchange markets through quite elevated Coskewness contain a better possibility of practicing a foremost loss than carry out currency exchange markets with relatively low Coskewness. By the mean of this, the existence of Coskewness constriction guides to slow down holdings of currency exchange markets through elevated Coskewness inside portfolio resolute by our replica, thus preventive problem hazard. Throughout era of marketplace growth with high volatilities (positively-skewed market returns), currency exchange markets amongst comparatively elevated Coskewness comprises a better possibility of producing great optimistic proceeds than perform currency exchange markets through moderately low Coskewness. In this case, the presence of the Coskewness constriction assists amplifies the assets of currency exchange markets with high Coskewness. This clarifies why imposing the Coskewness restraint increases the presentation of the elected portfolios.

Where μ_p and β_p are determined portfolio's normal profit and Coskewness, separately. There are various monetary ramifications of this model. Without Eq. (9), a financial specialist's portfolio choice depends exclusively on the attributes of money exchange market return and fluctuation. With Eq. (9), the speculator currently additionally considers a money trade market's capacity to support against unforeseen stuns to marketplace unpredictability that is estimated through its Coskewness.

As per Lemma 1, financial specialists lean toward money trade markets among affirmative Coskewness more than with negative Coskewness, in addition the oceans sets to a portfolio decreases the resulting portfolio's probability of outrageous negative proceeds and upgrades the probability of extraordinary optimistic proceeds. Coskewness, the depositor ought to set a negative β_p while the market skewness is negative. The depositor ought to set optimistic β_p but the marketplace skewness is affirmative. As a result, the effective portfolios controlled by our model (mean-change Coskewness (MVC) portfolios) are legitimately identified with economic situations. Naturally, during times of market down turn by means of elevated volatilities (the instance of adversely slanted marketplace proceeds) money trade markets with moderately high Coskewness have a more noteworthy possibility of encountering a significant misfortune than do cash trade markets with generally low Coskewness. For this situation, the nearness of the Coskewness limitation prompts low property of money trade markets with high Coskewness in the portfolio controlled by our model, accordingly restricting drawback chance. During times of market upswing with high volatilities (decidedly slanted market returns), cash trade markets with generally high Coskewness have a more noteworthy possibility of creating outrageous positive returns than do money trade markets with moderately low Coskewness. For this situation, the nearness of the Coskewness requirement helps increment the possessions of cash trade markets with high Coskewness, consequently better catching upside potential. This clarifies why forcing the Coskewness imperative improves the presentation of the chose portfolios.

Proposition1. If short-selling is acceptable, for some known anticipated profit μ_p and Coskewness β_p, there is a single result to our model for m≥2, the resolution is specified by:

$$v_p^* = \frac{(\mu_p - r_f)B_{22} - \beta_p z_2}{aB_{22} - z_2^2} bv_T + \frac{a\beta_p - z_2(\mu_p - r_f)}{aB_{22} - z_2^2} cv_s \quad (10)$$

Where E = (E(r)-1mr_f) , a = El**Σ**-1E, b =1ml**Σ**-1 E,$v_T = (\frac{1}{b})\Sigma^{-1}E$ c = 1ml**Σ**-1**Λ**,$v_s = (\frac{1}{c})\Sigma^{-1}\Lambda$, z_2 = El**Σ**-1**Λ**, and $B_{22} = \Lambda^l\Sigma^{-1}\Lambda$

Proof: Using Lag series proliferates pro the 2 restrictions in Problem (7) stated superior, we achieve the 1st order most constructive organize:

$$2\Sigma v_p - \lambda_1(E(r) - 1mr_f) - \lambda_2\Lambda = 0 \quad (10.1)$$

Where $\lambda_1\lambda_1$ and $\lambda_2\lambda_2$ are the Lag range proliferates. Resolving Eq. (10.1) for v_p yields:

$$v_p = \frac{\lambda_1}{2}\Sigma^{-1}(E(r) - 1mr_f) + \frac{\lambda_2}{2}\Lambda\Sigma^{-1} \quad (10.2)$$

Plugging Eq. (10.2) into Eq. (8) gives:

$$\mu_p = r_f + (\frac{\lambda_1}{2}\Sigma^{-1}(E(r) - 1mr_f) + \frac{\lambda_2}{2}\Lambda\Sigma^{-1})^l(r - 1mr_f) \quad (10.3)$$

Rearrange eq. (10.3) gives:

$$\mu_p - r_f = (\frac{\lambda_1}{2}\Sigma^{-1}(E(r) - 1mr_f) + \frac{\lambda_2}{2}\Sigma^{-1}\Lambda)^l(r - 1mr_f) \quad (10.4)$$

Plugging Eq. (10.2) into Eq. (9) gives:

$$\beta_p = (\frac{\lambda_1}{2}\Sigma^{-1}(E(r) - 1mr_f) + \frac{\lambda_2}{2}\Sigma^{-1}\Lambda)^l\Lambda \quad (10.5)$$

Rearranging Eq. (10.5) gives:

$$\beta_p = \frac{\lambda_1}{2}\Lambda^l\Sigma^{-1}(E(r) - 1mr_f) + \frac{\lambda_2}{2}\Lambda^l\Sigma^{-1}\Lambda \quad (10.6)$$

The subsequent scalars are different to accumulate simpler data:

E = (E(r)-1mr_f) , a = El**Σ**-1E, b =1ml**Σ**-1E, $v_T = (\frac{1}{b})\Sigma^{-1}E$, c=1ml**Σ**-1**Λ**,$v_s = (\frac{1}{c})\Sigma^{-1}\Lambda$

z_2 =El**Σ**-1**Λ**, and $B_{22} = \Lambda^l\Sigma^{-1}\Lambda$

Eqs. (10.4) and (10.6), we have:

$$\mu_p - r_f = \frac{\lambda_1}{2}a + \frac{\lambda_2}{2}z_2 \quad (10.7)$$

$$\beta_p = \frac{\lambda_1}{2}z_2 + \frac{\lambda_2}{2}B_{22} \quad (10.8)$$

Explain the two Lag rangian scalars $\frac{\lambda_1}{2}$ and $\frac{\lambda_2}{2}$ by means of society of Eqs. (10.7) and (10.8) are alternating these values into Eq. (10.2) yield the following:

$$v_p^* = \frac{(\mu_p - r_f)B_{22} - \beta_p z_2}{aB_{22} - z_2^2}\Sigma^{-1}(E(r) - 1mr_f) + \frac{a\beta_p - z_2(\mu_p - r_f)}{aB_{22} - z_2^2}\Sigma^{-1}\Lambda \quad (10.9)$$

The experienced portfolio for the particular credible revisit μ_p and Coskewness $\beta_p$in our model is:

$$v_p^* = \frac{(\mu_p - r_f)B_{22} - \beta_p z_2}{aB_{22} - z_2^2} bv_T + \frac{a\beta_p - z_2(\mu_p - r_f)}{aB_{22} - z_2^2} cv_s \quad (10.10)$$

Purpose 1 display that MV coefficient portfolios assure a three-fund panel theorem, anywhere three funds are the safe currency exchange market, tangency portfolio v_T resolute via mean-variance model, and portfolio v_s. The risk-free tangency portfolio is merely two funds in the two-fund partition theorem indoors the consistent mean-variance configuration. The masterwork of MVC capable portfolios involve to adding portfolio v_s to the usual ordered assortment make possible collect additional Coskewness limit, that's why hedging next to suffering market volatility. To recover hedging implement of portfolio v_s:

$$v_s = \frac{1}{s}\Sigma^{-1}\Lambda = \frac{1}{s}\Sigma^{-1}\frac{Cos(r, r_w)}{m_w^3} = \frac{1}{s}\Sigma^{-1}\frac{cov(r, h_w)}{m_w^3}, \quad (11)$$

Where $Cos(r, r_w) = (Cos\ (r_1, r_w), Cos(r_2, r_w), Cos\ (r_3, r_w), \dots. Cos(r_m, r_w))^1$,

$Cos\ (r, h_w) = (cov\ (r_1, h_w), cov\ (r_2, h_w), cov\ (r_3, h_w), \dots. cov\ (r_m, h_w))^1$

Eq. (11) shows the dimension of v_s replicate currency exchange market profits and promote instability growth equally, particular to h_w events the endorse volatility. Hence, adjustment in market instability can be held by portfolio v_s. Portfolio v_s is competent to provide security next to surprising events in market volatile scenery. When the marketplace is particularly impulsive, depositors can short market portfolio v_s to diminish the off-putting result of marketplace unpredictability on portfolio staging. So, portfolio v_s is signified as hedge portfolio.

Autoregressive Conditional Heteroscedasticity (ARCH) Model.

In the novel methodology of ARCH model, the volatility is predicted since an affecting mean of precedent error requisites, assume a random walk model:

$$Y_t = \alpha + Y_{t-1} + \varepsilon_t \qquad Y_t = \alpha + Y_{t-1} + \varepsilon_t \quad (12)$$

After first difference

$$\Delta Y_t = \alpha + \varepsilon_t \qquad \Delta Y_t = \alpha + \varepsilon_t \quad (13)$$

Where ΔY_t is the change in Y_t If Y_t is the log of variable (Y_t = log (Pt) Pt is the cost of a currency exchange market) ΔY_t is the proportion modify in the variable or could be the currency exchange market return. Fiscal resources similar to stock values frequently establish to go after accidental hike with flow. Stock prices enlarge by α per time, except are or else impulsive. It's hard to predict stock returns. Volatilities emerge in numerous spaces in finance linking to hazard: assortment, alternative penalty, pricing of financial imitative, VIX, Black-Schooled etc.

ARCH models (counting annex of them) are nearly all trendy models for financial instability. ARCH model narrates the discrepancy (variation) of the error ε_t. Inaccuracy variations are not regular: therefore we include Heteroscedasticity which financial records for the "H" in ARCH. The ARCH model with p lags is represented by ARCH (p). Present volatility is a mean of previous errors square:

$$\sigma_p^2 = \gamma_0 + \gamma_1\varepsilon_{t-1}^2 + \cdots \dots. + \gamma_p\varepsilon_{t-p}^2 \quad (14)$$

Where $\gamma_0, \gamma_1, \dots \dots \gamma_p$ is coefficient to be estimated?

Descriptive variables and dependent variable if have not any stock returns, Δy_t and then $\varepsilon_t = \Delta y_{t-1}$ in this case the ARCH model becomes: and volatility depends on recent values of Δy_t^2.

$$\sigma_p^2 = \gamma_0 + \gamma_1\Delta y_{t-1}^2 + \cdots \dots. + \gamma_p\Delta y_{t-p}^2 \quad (15)$$

Among many extension of ARCH model one most popular is GARCH. Most of the characteristics of ARCH and GARCH are related although GARCH is more flexible, much capable of alike an extensive diversity of patterns. Monetary time series frequently contain heavy extremity which means additional great returns. Significant properties of GARCH allow to efficiently modeling financial time series having obese conclusions. GARCH model with (p, q) lags are indicated by GARCH (p, q) having an instable equation form:

$$\sigma_p^2 = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \cdots \ldots + \gamma_p \varepsilon_{t-p}^2 + \lambda_1 \sigma_{t-1}^2 + \cdots \ldots + \lambda_p \sigma_{t-p}^2 \quad (16)$$

The most common shape is not regarded as Markovian model, as every sole previous error add to measure volatility. It provides sparing models that are amazing but not easy to assess and, yet in its most simple association, has confirmed dreadfully efficient in foreseeing dependent transformation. Nearly all generally utilized GARCH particular affirms that the best indicator of the variation in the subsequent time frame is a weighted ordinary quite a while before run normal alteration, the difference probable for this phase, such a refreshing standard is a clear-cut representation of flexible or knowledge conduct and can be considered of as Bayesian stimulating.

The significant currencies USD, GBP, EUR, AUD, CAD, QAR, SAR, KWD, AED, JPY and CNY have been obtained for construction portfolios. The Daily data is attained from https://uk.investing.com, www.foreignexchange.org.uk and https://www.investing.com from January 2004 to February 2020. We have assembled 10 currency exchange portfolios EW-UCJSAR, UEW-UCJSAR, EW-CS, UEW-CS, UEW-UPE, EW-UPE, EW-QSKA, UEW-QSKA, EW-UPEAC, and UEW-UPEAC. Five portfolios are equally weighted portfolios and five are unequally weighted portfolios. These portfolios are constructed according to the following formulas.

Empirical Analysis and Results. Table.1. represents outline figures of the statistics, we denote two groups of currencies portfolios for each, one is equally weighted and other is unequally weighted. Our outcomes illustrate that Coskewness is habitually statistically remarkable. The first spotlight on the collision on the model's costing inaccuracy and subsequent actions the predictable profit indirect through an alteration in Coskewness. Tab.1 represents conceptual figures of the data. We note down to CPEW-UPE as well as CPUEW-UPE give up the utmost yearly excess profits through a mean of more than 131 %, as CPUEW-CS makes a smaller amount than 23.7 % excess proceeds, the lowest amongst every one of currency portfolios. According to volatility, CPUEW-UPE and CPUEW-UPEAC currency portfolios reveal the maximum risk, while CPUEW-CS appeared designate secured between all of 10 currency portfolios.

Table 1

Standard deviation, Coskewness, Mean excess returns Currency Exchange Return relationship Portfolios for Pakistan

	$CP_{EW\text{-}UCJSAR}$	$CP_{UEW\text{-}UCJSAR}$	$CP_{EW\text{-}CS}$	$CP_{UEW\text{-}CS}$	$CP_{UEW\text{-}UPE}$	$CP_{EW\text{-}UPE}$	$CP_{EW\text{-}QSKA}$	$CP_{UEW\text{-}QSKA}$	$CP_{EW\text{-}UPEAC}$	$CP_{UEW\text{-}UPEAC}$
Mean returns	34.272	61.740	24.1196	23.708	131.685	134.76	92.30	51.22	78.64	98.10
Stedv	**4.877**	8.5182	2.7172	**2.5828**	**19.719**	20.123	26.85	**15.01**	22.801	**22.792**
Gamma	0.222	0.318	0.755	0.796	0.068	0.080	0.460	0.450	0.073	0.0739
Correlation										
$CP_{EW\text{-}UCJSAR}$	1.0000									
$CP_{UEW\text{-}UCJSAR}$	0.9974	1.0000								
$CP_{EW\text{-}CS}$	0.9499	0.9647	1.0000							
$CP_{UEW\text{-}CS}$	0.9356	0.9502	0.9968	1.0000						
$CP_{UEW\text{-}UPE}$	0.9852	0.9800	0.9306	0.9249	1.0000					
$CP_{EW\text{-}UPE}$	0.9832	0.9787	0.9322	0.9350	0.9998	1.0000				
$CP_{EW\text{-}QSKA}$	0.9695	0.9804	0.9588	0.9361	0.9358	0.9352	1.0000			
$CP_{UEW\text{-}QSKA}$	0.9717	0.9824	0.9596	0.9140	0.9384	0.93792	0.99971	1.0000		
$CP_{EW\text{-}UPEAC}$	0.9871	0.9833	0.9277	0.9134	0.9945	0.99418	0.95140	0.9535	1.0000	
$CP_{UEW\text{-}UPEAC}$	0.9888	0.9843	0.92718	0.9245	0.9948	0.99410	0.95090	0.9530	0.9997	1.0000

Source: author's calculations.

This chart reports the average surplus profits, standard deviations (Stedv), correlations of 10 Foreign Currencies in PAK assortments and the marketplace catalog, because the co-variances among proceeds on the currency portfolios and the quadrangle debased marketplace profits (Gamma). The Daily data from January 2004 to February 2020 is attained from https://uk.investing.com, www.foreignexchange.org.uk and https://www.investing.com websites. The following are currency portfolios definitions:

CPEW-UCJSAR (16.66%*CNY+16.66%*JPY+16.66%*SAR+16.66%*AED+16.66%*QAR+16.66%*USD)

CPUEW-UCJSAR (40%*(SAR+AED+QAR)+20%*(USD)+40%*(JPY+CNY)

CPEW-CS (50%*CNY+50%*SAR), CPUEW-CS (60%*CNY+40%*SAR)

CPUEW-UPE (40%*USD+30%*GBP+30%*EUR),

CPEW-UPE (33.3%*USD+33.3%*GBP+33.3%*EUR)

CPEW-QSKA (25%*QAR+25%*SAR+25%*KWD+25%*AED),

CPUEW-QSKA (40%*SAR+20%*QAR+10%*KWD+30%*AED)

CPEW-UPEAC (20%*USD+20%*GBP+20%*EUR+20%*AUD+20%*CAD),

CPUEW-UPEAC (30%*USD+20%*GBP+15%*EUR+20%*AUD+15%*CAD)

Furthermore, correlations between any pair of currency profits upper to 0.92 so, all portfolios are highly correlated. Figure.1. shows the matrix of correlation of all the currency exchange portfolios.

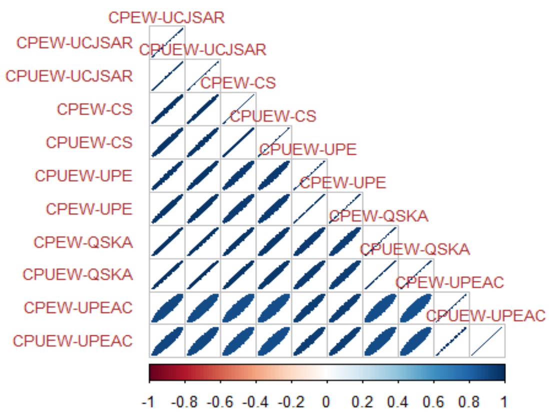

Fig. 1. Matrix representation of correlation among currency portfolios

Source: Created by the author.

Table. 1 gives details of efficient skewness (gamma) of a diversity of currency portfolios. Obviously, Gammas be non-zero and optimistic, representing to methodical skewness have fun significant position in explanation alterations in currency profits. In adding, CPUEW-UPE has the lowest gamma value (0.068), even as CPEW-QSKA has utmost gamma value (0.460). This recommends so as to CPUEW-UPE contribute low to high depressing market profits, even as the involvement of CPEW-QSKA to great negative market proceeds is the highest. By currency facts, we approximate all model constraint, after that plot the proficient boundaries in the mean-variance plane in different circumstances. Because the predictable marketplace skewness is negative, financiers can indicate depressing methodical skewness value ŋp consequent to an optimistic Coskewness.

Figure. 1 exhibits the traditional mean-variance proficient boundary and the professional boundaries established by our replica for different ŋp. Figure.2. illustrates, all the proficient borders with the logical skewness limitation are placed to the perfect of the conventional proficient leading edge, and the competent portfolios among the methodical skewness limit comprise a high variation as compared to proficient portfolio for the similar probable income.

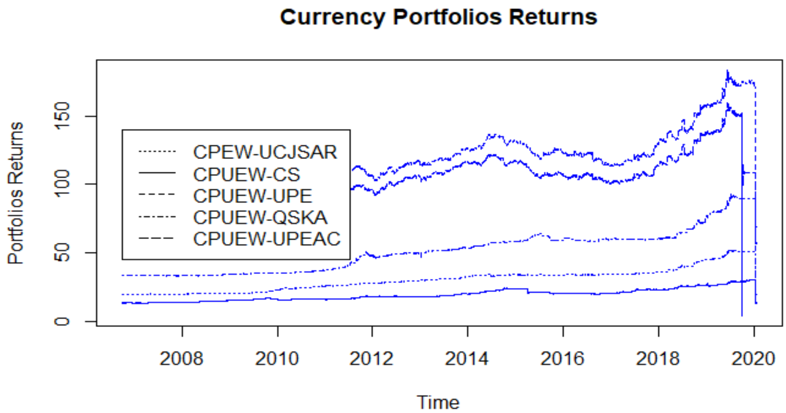

Fig. 2. Currency Portfolios Returns
Source: Created by the author.

Proficient outskirts among and lacking the methodical skewness limitation, this number designs the productive boondocks created with unsafe resources within the customary average-fluctuation model just while in the model through different efficient skewness imperatives. MV represents the mean-variance productive outskirts. ŋp speaks to the necessary orderly skewness in the model. MVP represents least fluctuation portfolio.

We propose for a particular the portfolio skewness diminishes through the intensity of necessary income. This proposes that financiers require trading small predictable profits for high skewness. It cannot be feasible to attain together a high proceeds and high skewness.

Connection of Coskewness with ARCH-GARCH Models.

Therefore, we first construct a time-varying risk premium coefficient incorporated GARCH-M model to study the time variation of the risk premium coefficient. On the basis of the above model, we propose a time-varying risk premium coefficient incorporated GARCH-M model with Harvey and Siddique's [15] autoregressive conditional skewness model to introduce the conditional skewness process. We employ time series daily data of currencies which has too much volatility in values skewness finds mean-return, risk-adjustment and gamma values to make easy for the selection of portfolio but for future forecasting of values different models are used such as ARIMA, ARCH and GARCH models. As currencies values having too much variation with each other so, through ARIMA models it is difficult to model the volatility of these time series values. But ARCH-GARCH model is very appropriate for modeling the volatility of these time series values with high difference. Modeling of five chosen equally weighted and unequally weighted currency portfolio is considered during ARCH-GARCH model to conclude the volatility.

Table 2

Modeling Equally Weighted Currency Portfolio (CP EW-UPEAC) Volatility through ARCH-GARCH

ARCH(1) Model using Currency Portfolio Return Data			
Variables	Coefficient Estimate	P-Value	95% Confidence Interval
Regression Equation Through ΔY as Reliant Patchy			
Seize	98.107	0.001	[83.081, 109.12]
ARCH Equation			
Capture	00.47	0.0000	[0.038, 00.056]
$\Delta\varepsilon_{t-1}^2$	0.8806	0.0000	[0.716, 0.9801]

Source: author's calculations.

Estimate of γ_1 is 0.8806, indicating that today volatility of currency portfolios depends strongly on the errors squared previous day.

Table 3

Modeling Equally Weighted Currency Portfolio (CPEW-QSKA) Volatility through ARCH-GARCH

ARCH(1) Model using Currency Portfolio Return Data			
Variables	*Coefficient Estimate*	*P-Value*	*95% Confidence Interval*
Regression Equation By ΔY as Needy Uneven			
Cut off	51.23	0.0012	[37.081, 62.129]
ARCH Equation			
Interrupt	1.537	0.0000	[1.026, 1.881]
$\Delta\varepsilon_{t-1}^2$	0.7313	0.0001	[0.673, 0.806]

Source: author's calculations.

Estimate of γ_1 is 0.7313, indicating that today volatility of currency portfolios depends strongly on the errors squared previous day.

Table 4.1

Modeling Equally Weighted Currency Portfolio (CPEW-UPE) Volatility through ARCH-GARCH

ARCH(1) Model using Currency Portfolio Return Data			
Variables	Coefficient Estimate	P-Value	95% Confidence Interval
Regression Equation By Way of ΔY as Reliant Erratic			
Capture	41.23	0.0021	[34.081, 47.73]
ARCH Equation			
Seize	1.537	0.0003	[1.016, 1.881]
$\Delta\varepsilon_{t-1}^2$	0.6311	0.0001	[0.5636, 0.6910]

Source: author's calculations.

Estimate of γ_1 is 0.6311, indicating that today volatility of currency portfolios depends strongly on the errors squared previous day.

Table 4.2

Modeling Equally Weighted Currency Portfolio (CPEW-CS) Volatility through ARCH-GARCH

ARCH(1) Model using Currency Portfolio Return Data			
Variables	Coefficient Estimate	P-Value	95% Confidence Hiatus
Regression Equation By Means of ΔY as Needy Changeable			
Seize	19.07	0.0021	[11.081, 24.13]
ARCH Equation			
Capture	00.37	0.007	[0.029, 0.431]
$\Delta\varepsilon_{t-1}^2$	0.6011	0.000	[0.4906, 0.7110]

Source: author's calculations.

Estimate of γ_1 is 0.6011, indicating that today volatility of currency portfolios depends strongly on the errors squared previous day.

Table 5

Modeling Equally Weighted Currency Portfolio (CP EW-UCJSAR) Volatility through ARCH-GARCH

ARCH(1) Model using Currency Portfolio Return Data			
Variables	Coefficient approximation	P-Value	95% Confidence intermission
Regression Equation Through ΔY as Reliant Unpredictable			
Interrupt	23.701	0.009	[17.301, 29.131]
ARCH Equation			
Intercept	00.301	0.000	[0.023, 00.036]
$\Delta\varepsilon_{t-1}^2$	0.6180	0.000	[0.546, 0.6710]

Source: author's calculations.

Estimate of γ_1 is 0.6180, indicating that today volatility of currency portfolios depends strongly on the errors squared previous day.

Significance of ARCH/GARCH. The ARCH/GARCH structures validate successfully in forecasting volatility alters. Practically, an extensive series of fiscal and profitable phenomenon reveal the collection of volatilities. We have observed ARCH/GARCH models explain the time progress of the standard size of squared inaccuracy, that is, the progression of the size of hesitation. Regardless of the observed achievement of ARCH/GARCH models, there is no actual consent on the financial motives why ambiguity inclined to gather. Therefore models perform better in several times and inferior in further stages. It is relatively easy to induce ARCH activities in virtual technique by construction suitable statements on negotiator manners. Such as, one can replicate ARCH performance in synthetic marketplaces by means of easy suppositions on mediator administrative procedures. The authentic monetary confront, conversely, is to clarify ARCH/GARCH performance in provisions of attributes of middle men actions with/otherwise financial variable that might be practically determined.

Density of five currency portfolios which are selected by us for investment purpose:

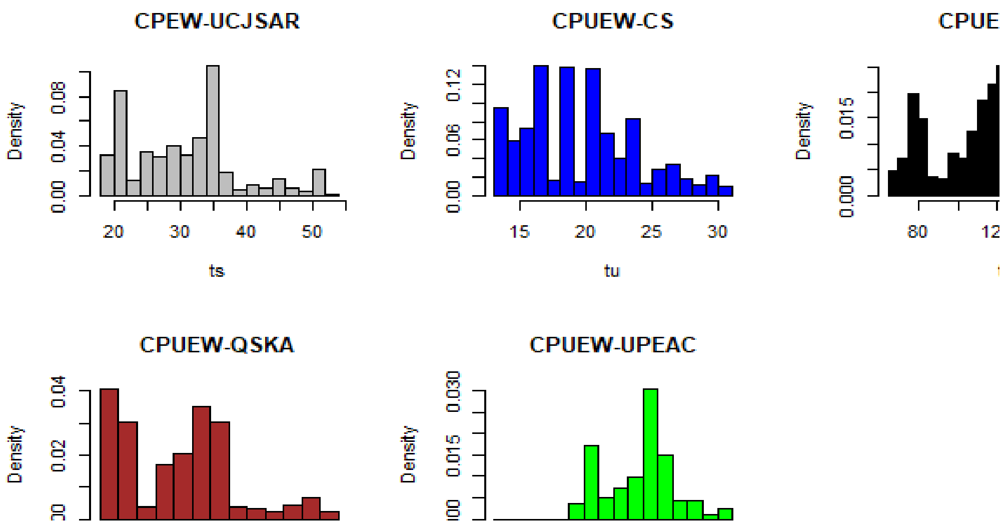

Fig. 3. Density of five currency portfolios

Source: Created by the author.

Figure.3. shows density of five currency portfolios selected from ten portfolios according to risk-adjustment and good return rate. The graph shows that the density of CP EW-UCJSAR 0.08 which shows that it is less risky currency portfolio, currency portfolio CPEW-QSKA also having very low density 0.04 and is less risky with better profitability return rate.

Conclusions. Our article approximates the risk of portfolio by means of Coskewness plus concerned ARCH-GARCH model for the optimization of currency portfolio. For the estimation of portfolio risk with additional accuracy, the paper employs Coskewness as risk calculator, average return and gives detail of efficient skewness (gamma) of a diversity of currency portfolios. This paper also applies ARCH-GARCH model, significant properties of GARCH allow to efficient modeling financial time series having obese conclusions. Then, we connect Coskewness with ARCH-GARCH models to optimize currency portfolio. Coskewness and ARCH-GARCH models consider equally the weight of currency portfolio proportions and variations of tail dependence connecting currency benefits. In conclusion, the outcomes of the basic research specify that it is extra proficient to optimize portfolio via ARCH-GARCH model instead of ARIMA model; the best portfolio is enhanced through Coskewness and ARCH-GARCH model. Coskewness and ARCH-GARCH model can be used in future research to optimize best portfolio with average return and less risk-adjusted from stock and currency portfolios.

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Мунieб Ахмад, Юсаф Али Хан, Иртаза Иштиак, Мухаммед Масуд. Использование асимметрии в постройке модели ARCH-GARCH при анализе валютных портфелей.

В работе используется соасимметрия как инструмент оценки риска и средних величин модели, а также для получения подробностей о параметрах эффективной асимметрии (гамма) для различных валютных портфелей. В этой работе применяется модель ARCH-GARCH, поскольку широкие возможности моделей типа GARCH позволяют эффективно моделировать финансовые временные ряды, имеющие существенные остатки. Затем соасимметрия была совмещена с моделью ARCH-GARCH для оптимизации валютного портфеля. В заключение проведено эмпирическое исследование десяти валютных портфелей пакистанского валютного рынка, в результате чего доказано, что соасимметрия может лучше охарактеризовать корректировки риска и среднюю дисперсию, а эффективность модели ARCH-GARCH для оптимизации валютного портфеля выше, чем модели ARIMA.

Ключевые слова: *валютный портфель, модель ARCH-GARCH, Пакистанская рупия, соасимметрия.*

SUPPORTING MATERIALS

Diagnostic Tests:

Jarque Bera Test

data: Residuals

X-squared = 2436679585, df = 2, p-value < 2.2e-16

Box-Ljung test

data: Squared.Residuals

X-squared = 24.201, df = 1, p-value = 8.68e-07

Step-by-step ARCH test (LM test)

Step-0ne

Time.reg <-lm(ts~1)

> summary(Time.reg)

Call: lm(formula = ts ~ 1)

Residuals:

Min	1Q	Median	3Q	Max
-93.829	-20.865	5.274	15.449	60.737

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 98.1072 0.3288 298.4 <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 22.92 on 4858 degrees of freedom

Step-Two

byd.ARCH <- lm(ehatsq~lag(ehatsq))

> summary(byd.ARCH)

Call: lm(formula = ehatsq ~ lag(ehatsq))

Residuals:

Min	1Q	Median	3Q	Max
-1.897e-12	-4.200e-14	-1.800e-14	1.400e-14	1.225e-10

Coefficients:

	Estimate	Std. Error	t value	Pr (>|t|)	
(Intercept)	-1.357e-12	3.255e-14	-4.169e+01	<2e-16	***
lag(ehatsq)	1.000e+00	3.911e-17	2.557e+16	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.76e-12 on 4857 degrees of freedom

Multiple R-squared: 1, Adjusted R-squared: 1

F-statistic: 6.538e+32 on 1 and 4857 DF, p-value: < 2.2e-16

LM-Test= 8.55

The result is the LM statistic, equal to 8.55, which is to be compared to the critical chi-squared value with α=0.05 and q=1 degrees of freedom; this value is ***χ2(0.95,1)=3.84****; this indicates that the null hypothesis is not rejected, concluding that the series has ARCH effects.*

Стаття надійшла до редколегії 30 серпня 2020 року